COMPARATIVE STUDY OF MODE MATCHING FORMULATIONS FOR MICROSTRIP DISCONTINUITY PROBLEMS†

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ABSTRACT

Several matrix formulations for the microstrip step discontinuity problem are compared. Although they are theoretically identical, one of them has a decided advantage in numberical labor, relative and absolute convergence. Results by this method are checked with other published data and with those independently derived by the modified residue calculus technique.

INTRODUCTION

This paper deals with the microstrip step discontinuity problem based on the waveguide model. Several papers on this subject (1,2,3) have presented numerical data. However, no detailed formulation method is given in these publications. The objective of the present paper is not to duplicate these data, but to place some foundations on how these data should be calculated. It has been known among researchers that numerical labor and accuracy depend on the choice of formulation even if several of theoretically identical formulations exist for a given problem. This is demonstrated in this paper.

The best formulation is decided based on the matrix size, relative and absolute convergence, and other numerical considerations. It turns out to be the one we often choose without clear reasoning. The data for a microstrip step discontinuity are compared with available data. They are also compared with the modified residue calculus technique which serves as an independent check of the numerical accuracy.

FORMULATION

The problem under study is the waveguide model for the microstrip step discontinuity shown in Figure 1(a). The structure is assumed to be symmetrical, and the parallel-plate waveguide is idealized with magnetic side-walls. For convenience of analyses, an auxiliary structure is introduced as in Figure 1(b). Only one half of the original structure is considered because of the symmetry, and the transversal magnetic wall at the discontinuity is recessed to create a new region C. The original structure is recovered by letting $d = 0$.

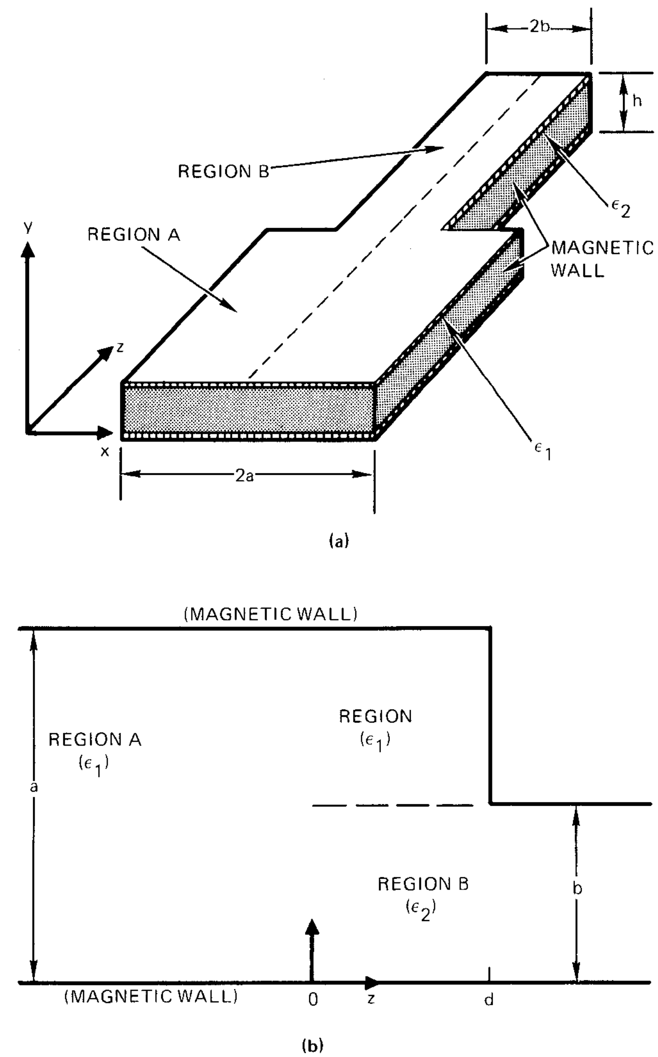

Fig. 1 Waveguide model for microstrip step discontinuity

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The mode-matching procedure begins with expanding the transverse electric and magnetic fields at the junction in terms of the normal modes on both sides of the junction.

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For TE_{no} (n = 0, 1, ...) excitation, we write down the E_y continuity equation:

$$\sum_{n=o}^{M}(A_n^{+}+A_n^{-})\phi_{an}= \begin{cases} \sum_{n=o}^{K}(B_n^{+}+B_n^{-})\phi_{bn} & o\le x\le b \\ \sum_{n=o}^{L} C_n\phi_{cn}(1+\rho_n) & b<x\le a \end{cases} \quad (1)$$

and a corresponding one for H_x.

In equations (1), ϕ_{an}, ϕ_{bn}, and ϕ_{cn} are normal modes in regions A, B, and C, respectively. An^+ and Bn^- are given incident field coefficients from regions A and B, while An^-, Bn^+, and Cn are the unknown excited field coefficients in regions A, B, and C, respectively, ρ_n is the reflection from the magnetic wall in region C.

From the modal orthogonality, we obtain the linear simultaneous equations for the unknown modal coefficients in matrix form:

$$\underset{\sim}{a}^{+}+\underset{\sim}{a}^{-}=G\bar{R}\underset{\sim}{d}^{+}+G\underset{\sim}{d}^{-} \quad (2)$$

$$Y_a(\underset{\sim}{a}^{+}-\underset{\sim}{a}^{-})=GY_d\bar{R}'\underset{\sim}{d}^{+}-GY_d\underset{\sim}{d} \quad (3)$$

$$G^T(\underset{\sim}{a}^{+}+\underset{\sim}{a}^{-})=\bar{R}\underset{\sim}{d}^{+}+\underset{\sim}{d}^{-} \quad (4)$$

$$G^TY_a(\underset{\sim}{a}^{+}-\underset{\sim}{a}^{-})=Y_d\bar{R}'\underset{\sim}{d}^{+}-Y_d\underset{\sim}{d}^{-} \quad (5)$$

Where $\underset{\sim}{a}^{+}(An^{+})$ and $\underset{\sim}{d}^{-}(Bn^{-}, O)$ are the excitation terms and $\underset{\sim}{a}^{-}(An^{-})$ and $\underset{\sim}{d}^{+}(Bn^{+}, Cn)$ are unknowns, Y_a and Y_d are diagonal matrices with the modal impedances as their diagonal elements. Matrix G collects the cross-coupling coefficients between the modal fields on both sides of the junction, while matrices $\bar{R}$ and $\bar{R}'$ contain the information on $(1+\rho_n)$ and $(1-\rho_n)$, respectively. All the matrices are square matrices of size (MxM): this requires that K + L = M.

When M→oo, we can prove that $G^{-1}\equiv G^T$. Therefore, equations (2) and (3) are equivalent to equations (4) and (5). Two independent vector equations are required to solve for two unknown vectors. Hence, for four pairs of equations, {(2),(3)}, {(4),(5)}, {(2),(5)}, and {(3),(4)}, substituting one equation into the other in the same pair, we have eight ways to solve for $\underset{\sim}{a}^{-}$ and $\underset{\sim}{d}^{+}$. They are define graphically in Figure 2. The approaches indicated by a solid arrow are classified as the formulations of the first kind and those indicated by a dashed arrow are of the second kind. Although the eight ways of solution are theoretically equivalent, their numerical behaviors are somewhat different, especially when the magnetic wall is introduced at the upper half of the junction ($d = 0, \rho_n = 1$).

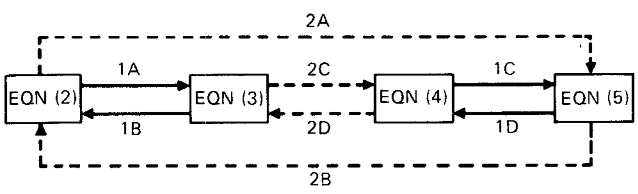

Fig. 2 Classification of formulations

For general cases $\rho_n \neq +1$, all the formulations require a matrix inversion of size (MxM). For our limiting case of d = 0, special modifications must be taken for some cases. Specifically, 1D and 2B need to invert a (M+L)x(M+L) matrix and 2C needs to invert a smaller (KxK) matrix. Hence, 2C is most attractive to us because of its potential of numerical efficiency. In the next section, we will examine the various approaches in terms of the numerical stability and convergence.

NUMERICAL RESULTS

We have chosen the structural parameters as: a=100, b=26.1, ϵ_1=2.2, ϵ_2=2.1. The reflection and transmission coefficients at the junction are calculated by varying the matrix size for different K/M ratios.

Since 1D and 2B have an apparent disadvantage in numerical calculations, they are not considered here. After extensive studies, we have found that 1A, 1B, and 1C are numerically identical. Similarly, 2A and 2D are numerically identical. Therefore, only three sets of data, corresponding to 1A, 2A, and 2C, are shown in Figure 3. It is observed that 2A and 2C suffer very little from the relative convergence problem. The problem is more serious in 1A; the result may converge to an incorrect value (4,5). The relative convergence effect can be more readily observed from the plot in Figure 4, showing the resultant transverse magnetic field at the junction for various K/L ratios.

A comparative study on the numerical efficiency for different approaches has also been done. In this case, L/K = 4, which is close to c/b, is chosen. The results are evaluated as a function of the matrix size required and shown in Figure 5. It is now obvious that 2C has definite advantages over other approaches. This formulation is to be used for further studies.

Let us refer back to equations (1) at this point. In many attempts, E_y in b<x<a region is not used as Hx=0 there. This choice turns out to be equal to our preferred choice.

To check the validity of our calculations, we have calculated the frequency response of a microstrip step discontinuity using the same parameters given by Kompa (3). The results are shown in Figure 6, which are in good agreement with Kompa's results. The small discrepancy is due to the different formulas used for obtaining the effective width and dielectric constant of the waveguide model. Furthermore, we have checked the results with those independently obtained by the modified residue-calculus technique. The results are shown in Table 1 for comparison. The calculations are performed using Kompa's parameters at 2 GHz. Here we have obtained an agreement down to the fourth decimal place.

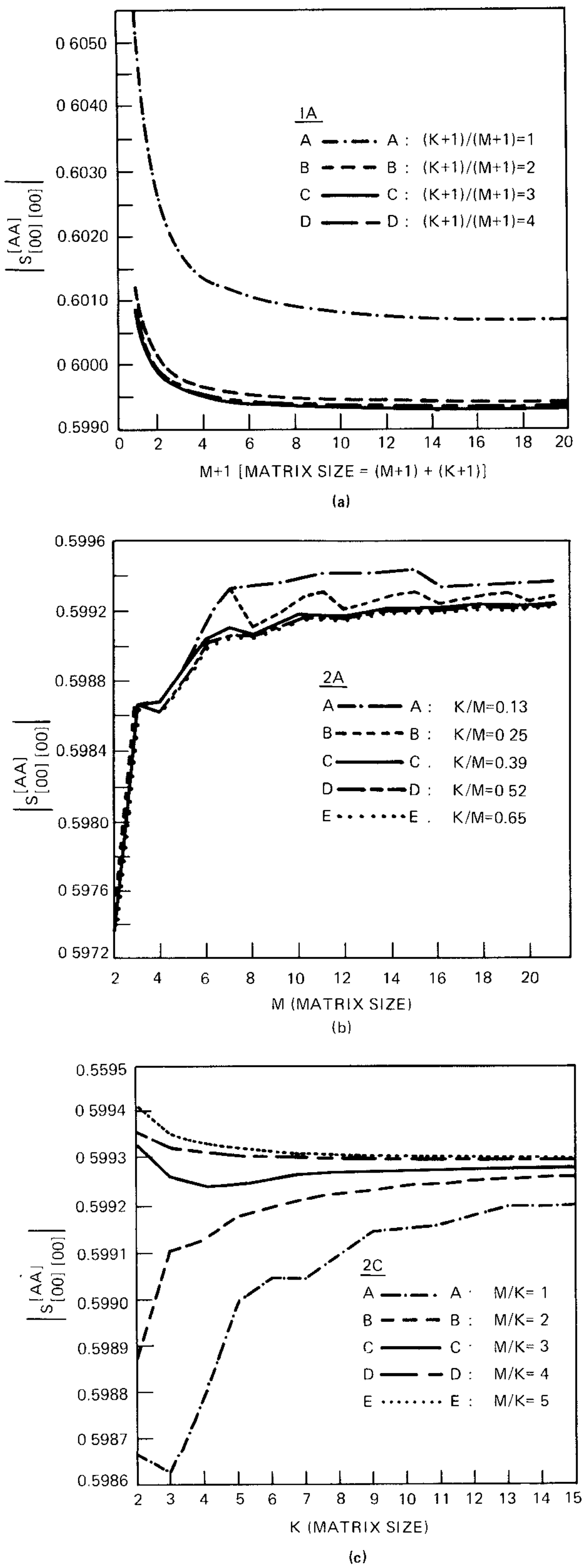

Fig. 3 Convergence study for various formulations. (The ordinate represents the magnitude of the reflection coefficient for the TEM mode in region A.)

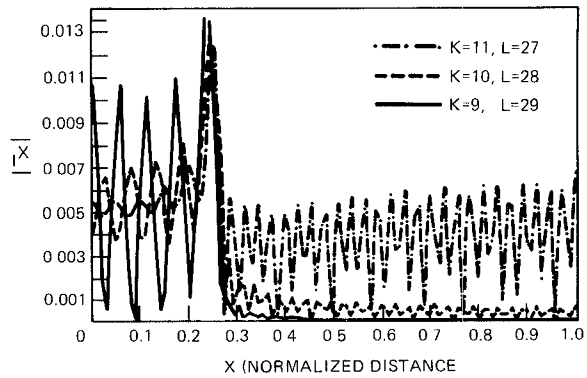

Fig. 4 Relative convergence problem demonstrated by field plots

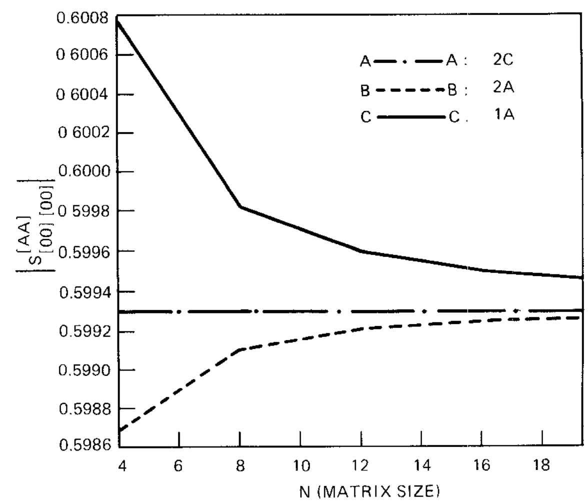

Fig. 5 Comparison of numerical efficiency

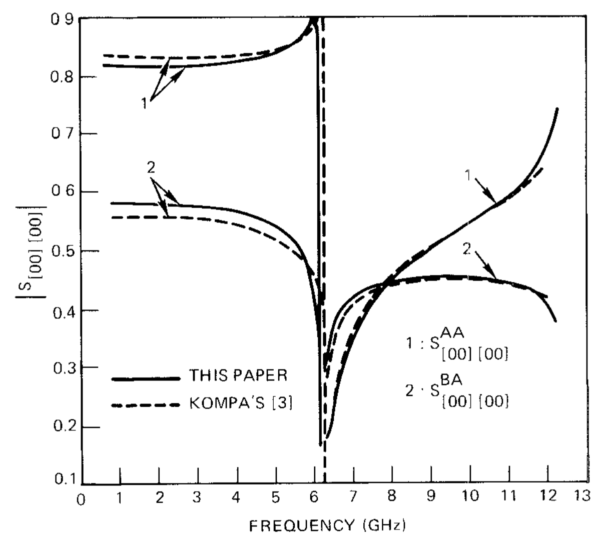

Fig. 6 Comparison with Kompa's results

TABLE I

Comparison of the results by mode-matching method and by modified residue calculus technique.

	Mode Matching	Modified Residue Calculus
$S^{AA}_{[00][00]}$	0.1837 - j 0.02291	0.1837 - j 0.02297
$S^{BA}_{[00][00]}$	-0.7856 - j 0.2227	-0.7855 + j 0.2233

CONCLUSIONS

The mode-matching method has been applied to analyze the microstrip step discontinuity problems based on the waveguide model. A comparison has been made among the various mode-matching solutions based on the matrix size, relative and absolute convergence. Although they are theoretically identical, one of them proves to be best suitable for numerical calculations. The results by this method are in good agreement with other published data and with those independently obtained by the modified residue calculus technique.

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